

These are only a few sample problems to *help* you prepare for the exam. You should also be certain that you completely understand the assigned homework, in-class work, and your class notes.

1. Is it possible that the terms of a series converge but the partial sums diverge?  
Can the partial sums converge but the terms diverge? Explain.

2. Do the following series diverge, converge conditionally, or converge absolutely?

$$(a) \sum_{n=1}^{\infty} (-1)^n \frac{n}{\pi n - 1} \quad (b) \sum_{r=10}^{\infty} \frac{(-1)^r}{r - 6} \quad (c) \sum_{k=1}^{\infty} (-1)^k \frac{2k}{k^3 + 2}$$

3. Find the interval of convergence for the power series

$$(a) \sum_{k=0}^{\infty} 3^k x^k \quad (b) \sum_{k=1}^{\infty} \frac{(x - 4)^k}{k 2^k}$$

4. Let  $a(x)$  be a continuous function that is positive and decreasing for all  $x \geq 1$ . Let  $a_k = a(k)$ . Rank the following values. Draw diagrams to explain your answer.

$$A = \sum_{k=5}^{n-1} a_k \quad B = \sum_{k=6}^n a_k \quad C = a_5 + \int_6^n a(x) dx \quad D = \int_5^n a(x) dx$$

5. Show that the following series converge and find a value  $N$  where  $S_N$  approximates the value of the series accurate within 0.001.

$$(a) \sum_{k=6}^{\infty} (-1)^k \frac{17}{k! + k^2} \quad (b) \sum_{k=1}^{\infty} 2k^2 e^{-k^2}$$

6. Calculate  $\int_0^1 \cos(x^2) dx$  accurate within 0.001 *by hand without the use of a calculator*.

7. Give examples that demonstrate why the Ratio Test is inconclusive when  $\lim_{k \rightarrow \infty} \frac{a_{k+1}}{a_k} = 1$ .

That is, give an example of a convergent series with this property and another example of a divergent series with this property.

8. Approximate  $\sqrt[3]{e}$  accurate within 0.001 by hand. *Hint:*  $\sqrt[3]{e} = e^{\frac{1}{3}}$