$$\Delta z = f(a+\Delta x, b+\Delta y) - f(a,b)$$

$$= \Big[f(a+\Delta x, b+\Delta y) - f(a, b+\Delta y) \Big] + \Big[f(a, b+\Delta y) - f(a,b) \Big]$$

$$= f_x(u, b+Dy)\Delta x + f_y(a,v)\Delta y$$

for some $u \in [a, a+\Delta x]$, $v \in [b, b+\Delta y]$

(MVT)

$$= f_x(a,b)\Delta x + \Big[f_x(u, b+\Delta y) - f_x(a,b) \Big] \Delta x + f_y(a,b)\Delta y + \Big[f_y(a,v) - f_y(a,b) \Big] \Delta y$$

$$= f_x(a,b)\Delta x + \epsilon_1 \Delta x + f_y(a,b)\Delta y + \epsilon_2 \Delta y$$

If $\epsilon_1 \to 0$ and $\epsilon_2 \to 0$ as $(\Delta x, \Delta y) \to (0,0)$, then

$$\Delta z = f(a+\Delta x, b+\Delta y) - f(a) \to dz = z - f(a),$$

where z is the point on the tangent plane above $(a+\Delta x, b+\Delta y)$, and so the points on the surface and on the tangent plane get closer and closer.

Note: If f_x and f_y are continuous in a neighborhood containing (a, b), then ϵ_1 and ϵ_2 *will* approach zero as $(\Delta x, \Delta y) \to (0, 0)$.