

Find the directions of maximum and minimum change of f at the given point, and the values of the maximum and minimum rates of change.

1. $f(x, y) = x^2 - y^3$, at the point $(-1, -2)$

$$\begin{aligned}\nabla f(x, y) &= \langle 2x, -3y^2 \rangle \\ \nabla f(-1, -2) &= \langle -2, -12 \rangle\end{aligned}$$

Thus the direction of maximum change of f at $(-1, -2)$ is in the direction of the vector $\langle -2, -12 \rangle$, and the value of this maximum increase is $\|\langle -2, -12 \rangle\| = \sqrt{4 + 144} = \sqrt{148}$.

The direction of minimum change (that is the biggest decrease) is in the direction of $\langle 2, 12 \rangle$, and the value of this greatest decrease is $-\sqrt{148}$.

2. $f(x, y, z) = 4x^2yz^3$, at the point $(1, 2, 1)$

$$\begin{aligned}\nabla f(x, y, z) &= \langle 8xyz^3, 4x^2z^3, 12x^2yz^2 \rangle \\ \nabla f(1, 2, 1) &= \langle 16, 4, 24 \rangle\end{aligned}$$

Thus the direction of greatest increase (maximum change) is in the direction of $\langle 16, 4, 24 \rangle$, with a value of $\sqrt{16^2 + 4^2 + 24^2} = \sqrt{848}$.

The direction of greatest decrease (minimum change) is in the direction of $\langle -16, -4, -24 \rangle$, with a value of $-\sqrt{848}$.