Let $\vec{\mathbf{a}} =< 3, 5 >$, $\vec{\mathbf{b}} =< 7, 2 >$, and $\vec{\mathbf{v}} =< 1, 3, -2 >$.

1. Find the projection of $\vec{\mathbf{a}}$ onto $\vec{\mathbf{b}}$

$$\begin{aligned}
\mathbf{proj}_{\vec{\mathbf{b}}}\vec{\mathbf{a}} &= \text{comp}_{\vec{\mathbf{b}}}\vec{\mathbf{a}}\left(\frac{\vec{\mathbf{b}}}{\|\vec{\mathbf{b}}\|}\right) \\
&= \left(\vec{\mathbf{a}} \cdot \frac{\vec{\mathbf{b}}}{\|\vec{\mathbf{b}}\|}\right)\left(\frac{\vec{\mathbf{b}}}{\|\vec{\mathbf{b}}\|}\right) = \left(\frac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{(\|\vec{\mathbf{b}}\|)^2}\right)\vec{\mathbf{b}} \\
&= \left(\frac{(3)(7) + (5)(2)}{\sqrt{7^2 + 2^2}^2}\right) < 7, 2 > \\
&= \left(\frac{31}{53}\right) < 7, 2 >= \left\langle \frac{217}{53}, \frac{62}{53} \right\rangle
\end{aligned}$$

2. Find two vectors perpendicular to $\vec{\mathbf{b}}$.

   In order to be perpendicular to $\vec{\mathbf{b}}$, the dot product must be zero.

$$< x, y > \cdot < 7, 2 >= 0 \Rightarrow 7x + 2y = 0.$$

   The vectors $< -2, 7 >$ and $< 2, -7 >$ seem like the most obvious choices.

   Any other choice will turn out to be a multiple of these two, since there are only two directions perpendicular to this vector.

3. Find three vectors perpendicular to $\vec{\mathbf{v}}$.

$$< x, y, z > \cdot < 1, 3, -2 >= 0 \Rightarrow 1x + 3y - 2z = 0.$$

   $< 1, 1, 2 >$ is the first that occurs to me, since $1+3 = 4$, and $4 = (2)(2)$.

   Also, $< -1, -1, 2 >$.

   But for something completely different, how about $< 3, 1, 3 >$?

   Notice: none of these three are multiples of any of the others. In space, there are an infinite number of directions perpendicular to any given vector.