

Example: Suppose that a cow is launched from a tower 15 meters off the ground at an angle of 30° from the horizontal with an initial speed of 60 meters per second. If the only force acting on the cow is gravity, how far will it travel? What is its maximum height? When will it hit the ground?

Translating this into a 2D vector problem (and remembering that gravity is $-9.8m/sec^2$), we are given that:

$$\vec{r}(0) = \langle 0, 15 \rangle, \quad \vec{v}(0) = 60 \langle \cos(\frac{\pi}{6}), \sin(\frac{\pi}{6}) \rangle, \quad \vec{a}(t) = \langle 0, -9.8 \rangle .$$

$$\vec{v}(t) = \int \vec{a}(t) dt \implies \vec{v}(t) = \langle 0, -9.8t \rangle + \vec{c}_1.$$

Since $\vec{v}(0) = \langle 30\sqrt{3}, 30 \rangle$, we get

$$\vec{v}(t) = \langle 30\sqrt{3}, -9.8t + 30 \rangle .$$

Similarly, $\vec{r}(t) = \int \vec{v}(t) dt = \langle 30\sqrt{3}t, -4.9t^2 + 30t \rangle + \vec{c}_2$. Using $\vec{r}(0) = \langle 0, 15 \rangle$, we get

$$\vec{r}(t) = \langle 30\sqrt{3}t, -4.9t^2 + 30t + 15 \rangle .$$

Our bovine will travel until the vertical component is 0, or $t = 6.587$ seconds. Since $\vec{r}(6.587) = \langle 342.279, 0 \rangle$, our cow travels 342.279 meters horizontally.

The maximum height happens when vertical velocity is 0, or when the y -component of $\vec{r}'(t)$ is zero. This happens at $t = 30/9.8 = 3.061$ sec. Since $\vec{r}(3.061224490) = \langle 159.06, 60.91 \rangle$, the max height is 60.91 meters.