1. Let $f(x) = \ln(x)$, $g(x) = x^2+3x$, and $h(x) = \cos(x)$. Find the following compositions and derivatives.

   (a) $(f \circ g)(x)$, $(f \circ g)'(x)$

   i. $(f \circ g)(x)$:

   $$(f \circ g)(x) = f\left(g(x)\right) = f(x^2+3x) = \ln(x^2+3x).$$

   ii. $(f \circ g)'(x)$:
   We know from the chain rule that

   $$\begin{aligned}(f \circ g)'(x) &= \frac{d}{dx} f(g(x))) \\ &= f'(g(x))g'(x) \\ &= \frac{1}{g(x)} \cdot (2x+3) \\ &= \frac{1}{x^2+3x} \cdot (2x+3)\end{aligned}$$

   (b) $(g \circ f)(x)$, $(g \circ f)'(x)$

   i. $(g \circ f)(x)$:

   $$(g \circ f)(x) = g\left(f(x)\right) = g(\ln(x)) = (\ln(x))^2 + 3(\ln(x)).$$

   ii. $(g \circ f)'(x)$:
   From the chain rule, we have

   $$\begin{aligned}(g \circ f)'(x) &= \frac{d}{dx}(g(f(x))) \\ &= g'(f(x))f'(x) \\ &= (2u+3) \cdot (\frac{1}{x}), \text{ where } u = f(x) \\ &= (2\ln(x)+3) \cdot (\frac{1}{x})\end{aligned}$$

   (c) $(h \circ g)(x)$, $(h \circ g)'(x)$

   i. $(h \circ g)(x)$:

   $$(h \circ g)(x) = h\left(g(x)\right) = h(x^2+3x) = \cos(x^2+3x).$$

ii. $(h \circ g)'(x)$
The chain rule tells us

$$\begin{aligned} (h \circ g)'(x) &= \frac{d}{dx}(h(g(x))) \\ &= h'(g(x))g'(x) \\ &= \sin(g(x))(2x+3) \\ &= \sin(x^2+3x)(2x+3) \end{aligned}$$

2. Find the derivatives of the following functions.

(a) $(3x^2+2)^{14}$

We can write this as $f(u(x))$, with $u(x) = 3x^2+2$ and $f(u) = u^{14}$. Therefore, the chain rule tells us that

$$\begin{aligned} \frac{d}{dx}(3x^2+2)^{14} &= f'(u)u'(x) \\ &= 14u^{13} \cdot (6x) \\ &= 14(3x^2+2)^{13}(6x) \end{aligned}$$

(b) $\ln(\sin x)$

Write this as $f(u(x))$, with $u(x) = \sin(x)$ and $f(u) = \ln(u)$. Therefore, the chain rule tells us that

$$\begin{aligned} \frac{d}{dx}(\ln(\sin(x))) &= f'(u)u'(x) \\ &= \frac{1}{u} \cdot \cos(x) \\ &= \frac{1}{\sin(x)}\cos(x) \\ &= \frac{\cos(x)}{\sin(x)} \\ &= \cot(x) \text{ (cotangent of x is defined to be cos(x)/sin(x).)} \end{aligned}$$

(c) $3\cos(\sqrt{x})$

Write this as $f(u(x))$, with $u(x) = \sqrt{x}$ and $f(u) = 3\cos(u)$. The chain rule says

$$\begin{aligned}\frac{d}{dx}(3\cos(\sqrt{x})) &= f'(u)u'(x) \\ &= -3\sin(u)\cdot\frac{d}{dx}(x^{1/2}) \\ &= -3\sin(\sqrt{x})\cdot\frac{1}{2}x^{-1/2} \\ &= -\frac{3}{2}\frac{\sin(\sqrt{x})}{\sqrt{x}}\end{aligned}$$

(d) $e^{(x^2)}$

Write this as $f(u(x))$, with $u(x) = x^2$ and $f(u) = e^u$.

If it helps you to see the outer and inner functions, use Maple notation for the exponential function:

$$e^{x^2} = \exp(x^2) \Longrightarrow u(x) = x^2, f(u) = \exp(u).$$

The chain rule says

$$\begin{aligned}\frac{d}{dx}(e^{(x^2)}) &= f'(u)u'(x) \\ &= e^u \cdot 2x \\ &= 2xe^{x^2}\end{aligned}$$

(e) $(e^x)^2$

- **Method 1:** Use exponential laws to rewrite this as e^{2x}. Then write this as $f(u(x))$, with $u(x) = 2x$ and $f(u) = e^u$. Then the chain rule tells us that

$$\frac{d}{dx}(e^{2x}) = f'(u)u'(x) = e^u \cdot 2 = 2e^{2x}.$$

- **Method 2:** Just proceed completely straightforwardly. Write $(e^x)^2$ as $f(u(x))$, with $u(x) = e^x$ and $f(u) = u^2$. Then using the chain rule,

$$\frac{d}{dx}((e^x)^2) = f'(u)u'(x) = 2u \cdot e^x = 2(e^x)(e^x) = 2e^{2x}.$$

(f) $\tan(3x^3 + 7x)$

Write this as $f(u(x))$ with $u(x) = 3x^3 + 7x$ and $f(u) = \tan(u)$. Then from the chain rule, we have

$$\frac{d}{dx}(\tan(3x^3+7x)) = f'(u)u'(x) = \sec^2(u)\cdot(9x^2+7) = (9x^2+7)\sec^2(3x^3+7x).$$

(g) $\cos(8^x)$

Write this as $f(u(x))$ with $u(x) = 8^x$ and $f(u) = \cos(u)$. Then the chain rule implies

$$\frac{d}{dx}(\cos(8^x)) = f'(u)u'(x) = -\sin(u)\cdot\ln(8)8^x = -\ln(8)8^x\sin(8^x).$$

(h) $8^{\cos(x)}$

Write this as $f(u(x))$ with $u(x) = \cos(x)$ and $f(u) = 8^u$. Then the chain rule gives us

$$\frac{d}{dx}(8^{\cos(x)}) = f'(u)u'(x) = \ln(8)8^u\cdot -\sin(x) = -\ln(8)\sin(x)8^{\cos(x)}.$$

(i) $\tan(\sin(3x))$

Write this as $f(u(x))$ with $u(x) = \sin(3x)$ and $f(u) = \tan(u)$. Then the chain rule tells us

$$\frac{d}{dx}(\tan(\sin(3x))) = f'(u)u'(x) = \sec^2(u)u'(x).$$

BUT $u(x)$ is itself a composition. We can do this without the chain rule, using some of our old rules, but just for practice:

Write $u(x) = \sin(3x)$ as $g(v(x))$ with $v(x) = 3x$ and $g(v) = \sin(v)$. Then the chain rule says

$$u'(x) = g'(v)v'(x) = \cos(v)\cdot 3 = 3\cos(3x).$$

Putting it all together,

$$\frac{d}{dx}(\tan(\sin(3x))) = \sec^2(u)u'(x) = \sec^2(\sin(3x))\cdot 3\cos(3x) = 3\sec^2(\sin(3x))\cos(3x).$$

(j) $e^{\cos(3x)}$

Write this as $f(u(x))$ with $u(x) = \cos(3x)$ and $f(u) = e^u$. Then by the chain rule,

$$\frac{d}{dx}(e^{\cos(3x)} = f'(u)u'(x) = e^u \cdot u'(x).$$

Write $u(x) = \cos(3x)$ as $g(v(x))$ with $v(x) = 3x$ and $g(v) = \cos(v)$. Then

$$u'(x) = g'(v)v'(x) = -\sin(v) \cdot 3 = -3\sin(3x).$$

Thus putting it all together,

$$\frac{d}{dx}(e^{\cos(3x)} = e^u \cdot u'(x) = e^{\cos(3x)} \cdot -3\sin(3x) = -3\sin(3x)e^{\cos(3x)}.$$

(k) $(\sin(3x^2))^2$

Write this as $f(u(x))$ with $u(x) = \sin(3x^2)$ and $f(u) = u^2$. Therefore, from the chain rule

$$\frac{d}{dx}((\sin(3x^2))^2) = f'(u)u'(x) = 2u \cdot u'(x).$$

Write $u(x) = \sin(3x^2)$ as $g(v(x))$ with $v(x) = 3x^2$ and $g(v) = \sin(v)$. The chain rule tells us that

$$u'(x) = g'(v)v'(x) = \cos(v) \cdot 6x = 6x\cos(3x^2).$$

Putting it all together,

$$\frac{d}{dx}((\sin(3x^2))^2) = 2u \cdot u'(x) = 2\sin(3x^2) \cdot 6x\cos(3x^2) = 12x\sin(3x^2)\cos(3x^2).$$

(l) $\sqrt{\ln(x^2+2x)}$

I can write this as $f(u(x))$, with $u(x) = \ln(x^2+2x)$ and $f(u) = \sqrt{u} = u^{1/2}$. Therefore the chain rule tells us that

$$\begin{aligned}
\frac{d}{dx}(\sqrt{\ln(x^2+2x)}) &= f'(u)u'(x) \\
&= \frac{1}{2}u^{-1/2} \cdot \frac{d}{dx}(\ln(x^2+2x)) - \text{Use the chain rule again!} \\
&= \frac{1}{2}(\ln(x^2+2x))^{-1/2} \cdot \frac{1}{x^2+2x} \cdot (2x+2)
\end{aligned}$$