

Let  $f(x) = \ln(x)$ ,  $g(x) = x^2 + 3x$ , and  $h(x) = \cos(x)$ . Find the following derivatives.

1.  $(f \circ g)'(x)$

We know from the chain rule that

$$\begin{aligned}(f \circ g)'(x) &= \frac{d}{dx}f(g(x)) \\ &= f'(g(x))g'(x) \\ &= \frac{1}{g(x)} \cdot (2x + 3) \\ &= \frac{1}{x^2 + 3x} \cdot (2x + 3)\end{aligned}$$

2.  $(g \circ f)'(x)$

From the chain rule, we have

$$\begin{aligned}(g \circ f)'(x) &= \frac{d}{dx}(g(f(x))) \\ &= g'(f(x))f'(x) \\ &= (2u + 3) \cdot \left(\frac{1}{x}\right), \text{ where } u = f(x) \\ &= (2\ln(x) + 3) \cdot \left(\frac{1}{x}\right)\end{aligned}$$

3.  $(h \circ g)'(x)$

The chain rule tells us

$$\begin{aligned}(h \circ g)'(x) &= \frac{d}{dx}(h(g(x))) \\ &= h'(g(x))g'(x) \\ &= \sin(g(x))(2x + 3) \\ &= \sin(x^2 + 3x)(2x + 3)\end{aligned}$$