1. Find $P_5(x)$, the fifth degree Taylor polynomial for $f(x)$ based at $x_0 = 1$.

   $P_5(x)$ will have the form

   $P_5(x) = a_0 + a_1(x-1) + a_2(x-1)^2 + a_3(x-1)^3 + a_4(x-1)^4 + a_5(x-1)^5$,

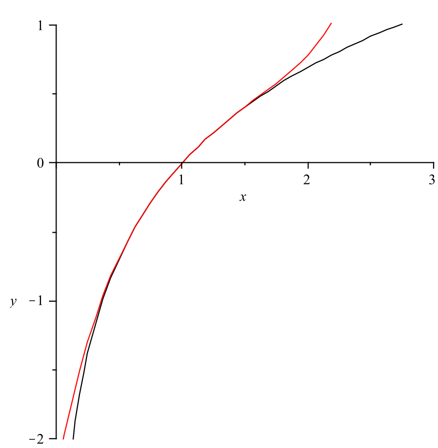
   but of course I still have to find a_k, $k = 0, 1, 2, 3, 4, 5$.

k	$f^{(k)}(x)$	$f^{(k)}(1)$	a_k
0	$\ln(x)$	0	0/0!=0/1=0
1	$\frac{1}{x} = x^{-1}$	1	1/1!=1
2	$-x^{-2} = -\frac{1}{x^2}$	-1	-1/2!=-1/2
3	$2x^{-3} = \frac{2}{x^3}$	2	$2/3! = 1/3$
4	$-3!x^{-4} = -\frac{3!}{x^4}$	$-3!$	$-3!/4! = -1/4$
5	$4!x^{-5} = \frac{4!}{x^5}$	$4!$	$4!/5! = 1/5$

   Thus we have

$$P_5(x) = 0 + 1(x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \frac{(x-1)^5}{5}.$$

2. Verify your answer by graphing $P_5(x)$ and $f(x)$ on the same set of axes.



3. Use $P_5(x)$ to find an approximation for $\ln(1.5)$.

$$\ln(1.5) \approx P_5(1.5) = .5 - \frac{.5^2}{2} + \frac{.5^3}{3} - \frac{.5^4}{4} + \frac{.5^5}{5} = .4073.$$

Compare this to the value Maple gives for $\ln(1.5)$:

$$\ln(1.5) = .4055.$$

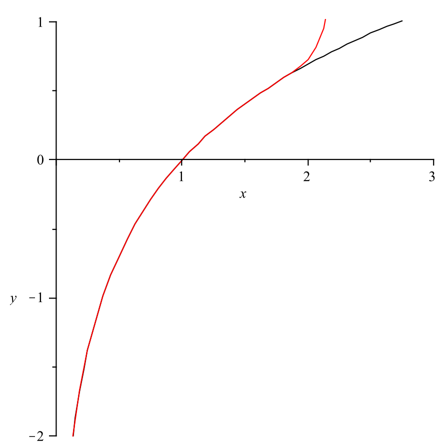
Not too shabby!

4. Now find $P_{15}(x)$.
Hint: You don't actually need to take all of the derivatives.

Looking at the pattern, it looks to me as if

$$P_{15}(x) = 0+1(x-1)-\frac{(x-1)^2}{2}+\frac{(x-1)^3}{3}-\frac{(x-1)^4}{4}+\frac{(x-1)^5}{5}-\frac{(x-1)^6}{6}+\frac{(x-1)^7}{7}-\ldots-\frac{(x-$$

I can always check this by looking at the graph:



Since it's fairly close, I can assume I extended the pattern correctly. However, you might be surprised at how little extra quality we gained with all those extra terms, after what we saw with $\cos(x)$ and how quickly its Taylor polynomial improves as an approximation. Adding extra terms to the Taylor polynomial always improves the quality of the approximation, but it varies from function to function how *much* it improves.