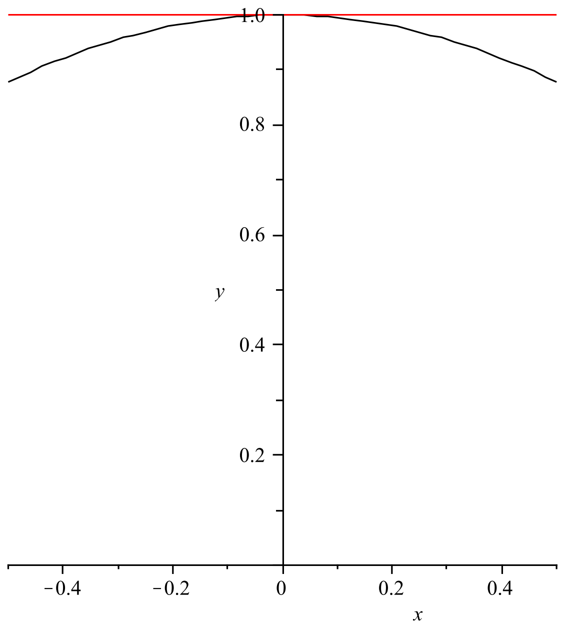
Consider a function and its tangent line:

Below is the graph of $\cos(x)$ (in black) and its tangent line (in red) at $x = 0$:

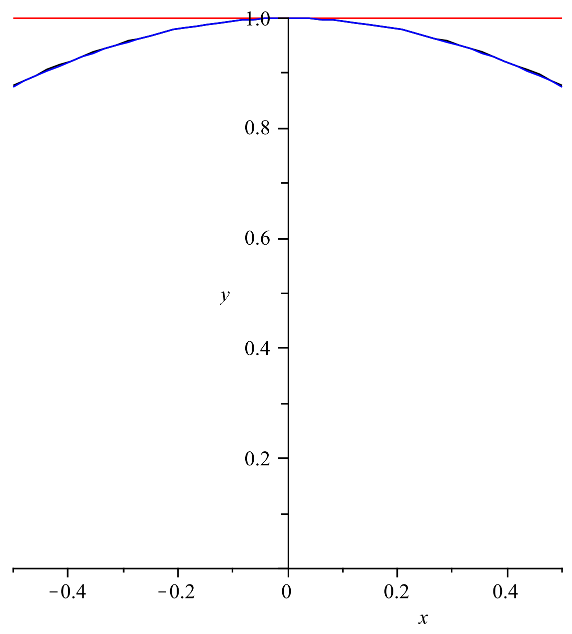


The tangent line gives a good approximation to the function near the point of tangency.

Why? Because they have the same slope. In other words, because they have the same first derivative at the point of tangency.

What if we make more derivatives agree?

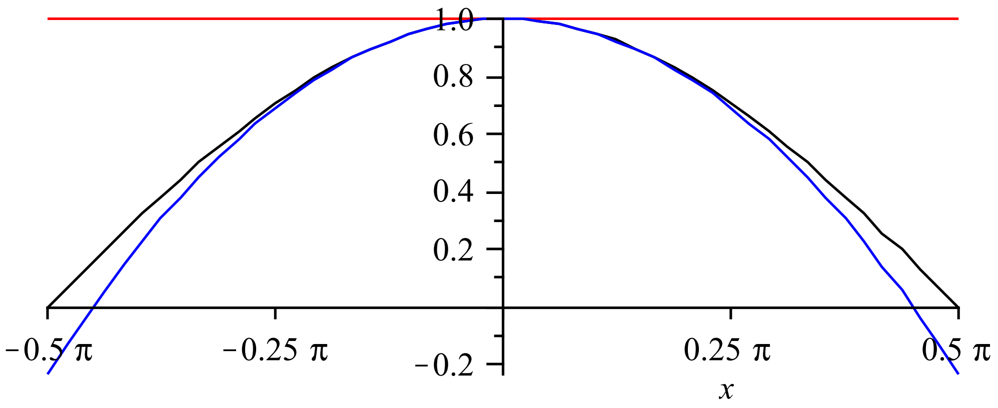
Below is the graph of $\cos(x)$ (in black), its tangent line (in red) at $x = 0$, and a new polynomial P_2 (in blue). At $x = 0$, $P_2(x)$ and $\cos(x)$ have the same y-value, the same slope, and the same concavity:



P_2 gives such a good approximation of $\cos(x)$ over this small interval, we can't even see the difference.

Why? Because I chose P_2 so that it has the same first and second derivative at $x = 0$.

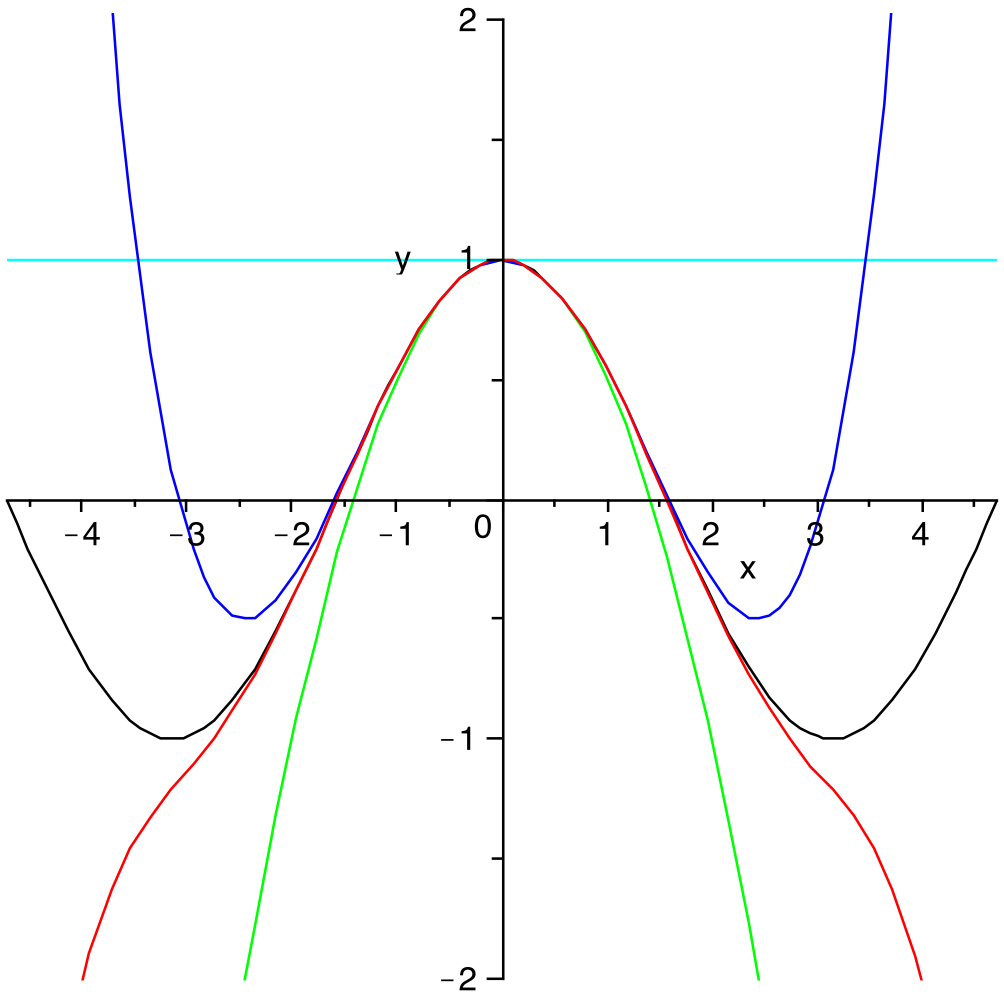
But if we look over a larger interval, we see that $P_2(x)$ starts to do a worse job of approximating $\cos(x)$ as we move farther away from $x = 0$.



How can we get a still better approximation?

The more derivatives our polynomial and our function agree on at that one point, the better job the polynomial does at approximating the function!

Here are the polynomials we've already seen, plus several more that match at more and more derivatives! (Our original function is the black one)



Let $f(x) = \sin(x)$ and
let $P_k(x)$ be the kth order Taylor polynomial for $f(x)$ at $x_0 = 0$.

1. Find $P_1(x)$, $P_2(x)$, $P_3(x)$, $P_4(x)$ and $P_5(x)$.

2. Verify your answer by graphing the polynomials and $f(x)$ on the same set of axes.

3. Use $P_5(x)$ to find an approximation for $\sin(3)$.

   Will this be larger or smaller than the actual value of $\sin(3)$?

4. Now find $P_{19}(x)$.
   Hint: You don't actually need to take all of the derivatives.