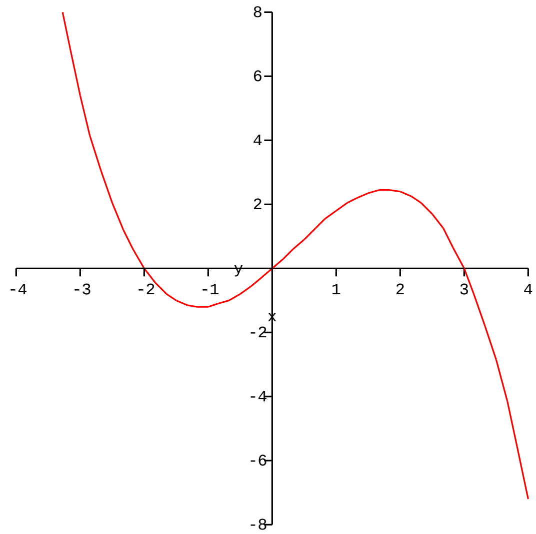
Graph of f''



1. Where is f concave up? concave down?

   f concave up where $f'' > 0$, so on $(-\infty, -2)$, $(0, 3)$.

   f concave down where $f'' < 0$, so on $(-2, 0)$, $(3, \infty)$.

2. Where does f have inflection points?

   f has inflection points where f changes concavity, so where f'' changes sign.

   At $x = -2$, $x = 0$, and $x = 3$.

3. Suppose that $f'(-1) = 0$ and $f'(1) = 0$. If possible, classify $x = -1$ and $x = 1$ as local maxima or local minima of f.

   - Because $f'(-1) = 0$, I know that f is flat there. It has a local max, local min, or inflection point there.
     Since $f''(-1) < 0$, f is concave down there. There's only one way a function can be flat and concave down – the function must have a maximum there!

- f also must have a local max, local min, or inflection point at $x = 1$. This time, $f'' > 0$, so f is concave up, so f has a local min.

4. Suppose that $f'(0) = 0$. Is f increasing or decreasing at $x = 1$? at $x = -1$?

   - First of all, since f'' changes sign at $x = 0$, we just have an inflection point at $x = 0$. Not really relevant, I just wanted to mention it.
   - On $[0, 1]$, $f'' > 0$. This means that $f'$is increasing on$[0, 1]$. Since f' begins that interval at 0, and increases from there, $f'(1) > 0$.

     Therefore f is increasing at $x = 1$.
   - On $[-1, 0]$, $f'' < 0$. This means that f' is decreasing on $[-1, 0]$. Since f' *ends* that interval at 0, and decreases to get there, we must have $f'(-1) > 0$.

     Therefore, f is increasing at $x = -1$ as well.

5. Suppose that $f'(-1) = -2$ and $f(-1) = 2$. Could $f(0) = 3$?
   (*Hint:* Can you determine if f is increasing or decreasing on $[-1, 0]$?)

   On $[-1, 0]$, $f'' < 0$, and so $f''$is decreasing. Since it begins the interval at -2, and decreases from there, $f'(0) < 0$, and f' is negative on the interval $[-1, 0]$.

   Thus f is decreasing throughout the interval $[-1, 0]$. Since $f(-1) = 2$, and f is decreasing all the way to get to $x = 0$, $f(0)$ can not be 3!