Derivatives we know so far

$f(x)$	$f'(x)$	Antiderivative $F(x)$
$f(x) = x^n$	$f'(x) = nx^{n-1}$	$F(x) = \frac{x^{n+1}}{n+1}$
$f(x) = e^x$	$f'(x) = e^x$	$F(x) = e^x$
$f(x) = b^x$	$f'(x) = \ln(b)b^x$	$F(x) = \frac{1}{\ln(b)}b^x$
$f(x) = \ln(x)$	$f'(x) = \frac{1}{x}$	$F(x) =???$
$f(x) = \log_b(x)$	$f'(x) = \frac{1}{\ln(b)}\frac{1}{x}$	$F(x) =???$
$f(x) = \sin(x)$	$f'(x) = \cos(x)$	$F(x) = -\cos(x)$
$f(x) = \cos(x)$	$f'(x) = -\sin(x)$	$F(x) = \sin(x)$

We also know:

For any function $f(x)$ and constant a,

Rule	Example
$[f(x+a)]' = f'(x+a)$	$\frac{d}{dx}(\ln(x+12)) = \frac{1}{x+12}$
$[f(x)+a]' = f'(x)$	$\frac{d}{dx}(\cos(x)+\pi) = -\sin(x)$
$[af(x)]' = af'(x)$	$\frac{d}{dx}(3\sin(x)) = 3\cos(x)$
$[f(ax)]' = af'(ax)$	$\frac{d}{dx}(e^{27x}) = 27e^{27x}$

Why the product rule is true

$$= \frac{d}{dx}\left(f(x)\ g(x)\right)$$

$$= \lim_{h\to 0} \frac{f(x+h)\ g(x+h) - f(x)\ g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)\ g(x+h) - f(x)\ g(x+h) + f(x)\ g(x+h) - f(x)\ g(x)}{h}$$

$$= \lim_{h\to 0} \left(\frac{f(x+h)\ g(x+h) - f(x)\ g(x+h)}{h} + \frac{f(x)\ g(x+h) - f(x)\ g(x)}{h} \right)$$

$$= \lim_{h\to 0} \left(\left[\frac{f(x+h) - f(x)}{h} \right] g(x+h)\ +\ f(x) \left[\frac{g(x+h) - g(x)}{h} \right] \right)$$

$$= f'(x)g(x) + f(x)g'(x)$$