Let

$$f(x) = \cos(x) - \sin(x)$$
$$g(x) = 4e^x - 3\cos(x+1) - \tfrac{1}{x}$$
$$h(x) = 3\sin(4) + 2\sin(3x) - \ln(x) + x^{732}$$

1. Find the derivatives of each function.

   (a) Since $\frac{d}{dx}(\cos(x)) = -\sin(x)$ and $\frac{d}{dx}(\sin(x)) = \cos(x)$, we have

   $$f'(x) = -\sin(x) - \cos(x).$$

   (b) I'll go through finding $g'(x)$ piece by piece.

   - $\frac{d}{dx}(4e^x) = 4e^x$
   - To find $\frac{d}{dx}(\cos(x+1))$, remember: a horizontal shift doesn't change the shape, it just shifts it. So the slopes don't change, except for a horizontal shift by the same amount.
     In short, according to Theorem 4 from Section 2.2,

     $$\frac{d}{dx}(s(x+a)) = s'(x+a).$$

     Thus to find $\frac{d}{dx}(\cos(x+1))$, I just plug $x+1$ into the derivative of $s(x) = \cos(x)$. $s'(x) = -\sin(x)$, so

     $$\frac{d}{dx}(cos(x+1)) = -\sin(x+1).$$

   - Rewrite $\frac{1}{x}$ as x^{-1}.
     Thus $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}(x^{-1}) = -x^{-2}$.

   Putting all of that together,

   $$g'(x) = 4e^x + 3\sin(x+1) + x^{-2}.$$

(c) Again, take it piece by piece

- $3\sin(4)$ is just a number, so $\frac{d}{dx}(3\sin(4)) = 0$.
- $2\sin(3x)$ is the same as $2\sin(x)$, except that it's compressed by a factor of 3. That means that it's changing 3 times as fast, so since $\frac{d}{dx}(2\sin(x)) = 2\cos(x)$, we must have

$$\frac{d}{dx}(2\sin(3x)) = 3 \cdot 2\cos(3x) = 6\cos(3x).$$

(Theorem 4 of Section 2.2 says $\frac{d}{dx}(f(kx)) = kf(kx)$)

- $\frac{d}{dx}(\ln(x)) = 1/x$.
- $\frac{d}{dx}(x^{732}) = 732x^{731}$.

Thus

$$h'(x) = 0 + 6\cos(3x) - 1/x + 732x^{731}.$$

2. Now find the antiderivative of each function.
Check your answer by taking the derivative!

(a) What would I differentiate to get $f(x) = \cos(x) - \sin(x)$?
I know that $\frac{d}{dx}(\sin(x)) = \cos(x)$ and $\frac{d}{dx}(\cos(x)) = -\sin(x)$.
That means that I would differentiate $\sin(x)$ to get $\cos(x)$, so that gives me the first half.
What would I differentiate to get $-\sin(x)$? I'd differentiate $\cos(x)$, of course. So that gives the second part, and

$$F(x) = \sin(x) + \cos(x).$$

(b) What would I differentiate to get $4e^x - 3\cos(x+1) - \frac{1}{x}$?
Well, we've already seen that $\frac{d}{dx}(4e^x) = 4e^x$, so we know that we'd differentiate $4e^x$ to get $4e^x$.
What would we differentiate to get $3\cos(x+1)$? Based on what we found when differentiating, it makes sense that we'd differentiate $3\sin(x+1)$ to get $3\cos(x+1)$.

As for $\frac{1}{x}$, it doesn't help us when antidifferentiating this particular power of x to use the power rule, as we'd just end up with 0 in the denominator. Fortunately, we now know that the antiderivative of $\frac{1}{x}$ is $\ln(x)$. Thus

$$G(x) = 4e^x - 3\sin(x+1) - \ln(x).$$

(c) What would I differentiate to get $3\sin(4)+2\sin(3x)-\ln(x)+x^{732}$? Again, let's just go through it piece by piece.

- $3\sin(4)$ is just a constant m. I know that the only types of functions that have constant slope are lines – and if the slope is m, the line has the form $mx+b$ – but we don't worry about those pesky additive constants here. So an antiderivative of $3\sin(4)$ is just $3\sin(4)\cdot x$.
- When we differentiated $2\sin(3x)$, we got $3\cdot 2\cos(3x)$. This time we're antidifferentiating. What do we get? Just go ahead and guess, and then differentiate the result!
  Let's try $2\cos(3x)$. The derivative of this is $-3\cdot 2\sin(3x)$, when I was aiming for $2\sin(3x)$. That means I'm off by a negative sign *and* my result is 3 times too big.
  So I try $-\frac{2}{3}\cos(3x)$. The derivative of this is $-3\cdot -\frac{2}{3}\sin(3x) = 2\sin(3x)$, which is exactly what I'm aiming for, so my second try was right!
- As for what an antiderivative of $\ln(x)$ is – you've never seen anything that when you antidifferentiate it, you get $\ln(x)$. So ... we can't do that part yet. There's still a lot more to learn, apparently!
- Skipping over that gaping hole in our result, we get to the antiderivative of x^{732}. That's just the same as it's been for a couple weeks – add 1 to the power and divide by the new power, so an antiderivative would be $x^{733}/733$.

Thus

$$H(x) = 3\sin(4)x - \frac{2}{3}\cos(3x) - \boxed{?} + x^{733}/733.$$

3. Find the maximum and minimum values of $f(x)$ on the interval $[-\pi, \pi]$.

   The maximum and minimum values of $f(x)$ can occur at stationary points, or at the endpoints.

   To find the stationary points, I need to take the derivative, set it equal to zero, and solve for x.

   Stationary points:

$$-\sin(x) - \cos(x) = 0 \Rightarrow \sin(x) = -\cos(x) \Rightarrow x = 3\pi/4, -\pi/4.$$

   Thus the only possible places the maximum and minimum value of $f(x)$ could occur are the stationary points ($x = 3\pi/4$, $x = -\pi/4$) and the endpoints ($x = -\pi$, $x = \pi$).

x	$f(x) = \cos(x) - \sin(x)$
$-\pi$	$\cos(-\pi) - \sin(-\pi) = -1$
$-\pi/4$	$\cos(-\pi/4) - \sin(-\pi/4) = \sqrt{2}/2 - (-\sqrt{2}/2) = \sqrt{2}$
$3\pi/4$	$\cos(3\pi/4) - \sin(3\pi/4) = -\sqrt{2}/2 - (\sqrt{2}/2) = -\sqrt{2}$
π	$\cos(\pi) - \sin(\pi) = -1$

   Thus the minimum value of $f(x)$ on $[-\pi, \pi]$ is $-\sqrt{2}$, and it occurs at $x = 3\pi/4$; the maximum value is $\sqrt{2}$, which occurs at $x = \pi/4$.